ALLTYPES in the Web

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ALLTYPES abbreviates *ALgebraic Language and TYPe System.* It is a computer algebra type system with particular emphasis on differential algebra and differential equations; it is available in the internet. This includes a complete documentation and an interactive environment for working with the system; as a consequence, no installation work and updates are necessary. Access to the ALLTYPES system may be obtained by online registration on the website `www.alltypes.de` without charge. In this document a short description of the most important features of ALLTYPES is provided.

The ALLTYPES Online Documentation. Upon registration, the user may reach the start page entitled

`ALLTYPES Working Environment`

This is the entry point for both the documentation section and the interactive user interface. The new user is strongly advised to follow the suggestions given in the `README` window first.

The documentation page is subdivided into two major parts. The large part at the right guides the user through some general system information. It covers several topics and subtopics that are ordered from top to bottom by increasing familiarity with the system.

`Getting Started with ALLTYPES.` Contains some information concerning the functionality of the buttons under the label `System Usage` in the left part of this page. The `Start ALLTYPES` button activates an interactive ALLTYPES session in a new popup window. The `Contact` button opens a popup window from which messages may be sent to the system operator, e.g. error messages, suggestions for new functions etc.

`Becoming Familiar with ALLTYPES.` This part explains the functionality of the collected buttons under the label `System Documentation` at the left. In the first place, this are the `Demos` and `Tutorials` buttons; they provide short descriptions of the respective ALLTYPES sessions. Clicking the `Type System` button, a complete listing of the currently supported types is shown.

`Solving Problems with ALLTYPES.` The most important functionality of this website is explained here. It describes the `User Interface Functions` pop-up menues in the left part; they are organized by three subjects: commutative algebra, differential algebra and differential equations.

`Reading and Searching.` Gives a short description of the pop-up menues of the `Data Base` and `Literature` buttons. The former contains a large part of the collection of differential equations in the book by Kamke. The literature shown in the latter displays publications that have been used for implementing ALLTYPES; it is organized by authors.

The ALLTYPES Interactive Environment. In order to work with the ALLTYPES system, the interactive window has to be opened by clicking on the link `Start ALLTYPES` at the top left. Now the userfunctions of the ALLTYPES system are available for interactive computations. Input is submitted in the algebraic mode language of REDUCE which is the implementation language of ALLTYPES. It has a Pascal like syntax similar to Maple or Mathematica, extended by a type manipulation language that is enclosed by a pair of bars |...|. Any such type instruction, following an input in the algebraic language, converts it into a mathematical object of the assigned type that is accepted by the ALLTYPES user functions.

In order to become familiar with the system the user should run the demos first. After an interactive session has been initiated by clicking the button `Start Alltypes`, any demo may be started by submitting one of the following inputs.

`Demo Commalg;` or `Demo Diffalg;` or `Demo Diffeqs;`

As the names of these demos suggest, they deal mainly with problems from commutative algebra, differential algebra or differential equations respectively. They do not require any user action, the only input necessary is a "y" or "no" whenever the query `cont?` appears; submitting "cont;" the session is continued. This is different with the tutorials where user interaction is explicitly encouraged. At first the tutorial

`Tutorial Basic;`

should be run. It provides the necessary familiarity with basic system features like type manipulations. In addition there are three tutorials with the same name as the demos. They may be started by entering the following input.

```
Tutorial Commalg;  or  Tutorial Diffalg;  or  Tutorial Diffeqs;
```

The same remarks apply to the topics of these three tutorials as above to the respective demos.

When these activities are completed the user should be ready for solving problems of his own by applying the user interface functions. For this purpose the documentations of the user interface functions are essential. Each of the three sections comprises between one or two dozen functions, they are listed in the pop-up menue below the corresponding entry at the left. Clicking on any of them, a description of the respective function appears in the main part of the website in a standardized form.

As explained previously, the type information for the arguments of any user function is crucial. Functions with identical names and different argument types are considered as different functions. A complete listing of all currently available types may be obtained by clicking on the link `Type System` in the left part of the main page. Type assignments are achieved as explained above. The specification explains shortly in mathematical terms what the function does. To this end it may be helpful sometimes to consult the given literature. Often the examples provide further insight. They may be reproduced if they are submitted as input to the interactive user interface. It may be instructive to apply small variations of the given examples and study its effect on the output. A good source of data for input are the differential equations listed in the `Data Base` menue in the left part. By cut-and-paste they may be easily submitted as input to the interactive user interface! .

An Interactive ALLTYPES Session. It is described now how various problems in differential algebra are solved by means of the ALLTYPES userfunctions. The inputs are given in typewriter fonts; the output is displayed in a frame exactly as it appears in the interactive session.

A typical problem is the computation of a Janet basis, the differential analogue of a Gröbner basis in commutative algebra. In the ring of differential operators $\mathbb{Q}(x,y)[\partial_x,\partial_y]$ the following left ideal is defined.

```
I:={D(x,2)+D(x,y)+(x+y)/(x*y)*D(x)+1/x*D(y)+(x-y)/(x**2*y),
    D(x,2)+D(y,2)+1/x*D(x)+1/y*D(y),
    D(x,y)-D(y,2)+1/y*D(x)-(x-y)/(x*y)*D(y)+(x+y)/(x*y**2)};
```

with the obvious notation $\mathtt{D(x)} \equiv \partial_x$, $\mathtt{D(x,y)} \equiv \partial_{xy}$ etc. It may be displayed by

```
Show(I|LDOID(RATF Q,x,y,GRLEX)|);
```

$$\langle \partial_{xx}+\partial_{xy}+\frac{x+y}{xy}\partial_x+\frac{x-y}{x^2y},\quad \partial_{xx}+\partial_{yy}+\frac{1}{x}\partial_x+\frac{1}{y}\partial_y-\frac{x^2+y^2}{x^2y^2},\quad \partial_{xy}-\partial_{yy}+\frac{1}{y}\partial_x-\frac{x-y}{xy}\partial_y+\frac{x+y}{xy^2}\rangle$$

The type information enclosed by a pair of |...| bars following the `I` in the input should be particularly observed. A Janet basis representation is obtained by calling

```
JanetBasis(I|LDOID(RATF Q,{x,y},GRLEX)|);
```

$$\langle \partial_{yy}+\frac{1}{y}\partial_y-\frac{1}{y^2},\partial_{xy}+\frac{1}{y}\partial_x+\frac{1}{x}\partial_y+\frac{1}{xy},\partial_{xx}+\frac{1}{x}\partial_x-\frac{1}{x^2}\rangle$$

Term order: $GRLEX, x \succ y$

The Loewy decomposition of I is obtained by calling

```
LoewyDecomposition(I|LDOID(RATF Q,x,y,GRLEX)|);
```

Loewy decomposition:

$$Lclm(\langle\partial_y-\frac{1}{y},\partial_x+\frac{1}{x}\rangle,\langle\partial_y+\frac{1}{y},\partial_x-\frac{1}{x}\rangle,\langle\partial_y+\frac{1}{y},\partial_x+\frac{1}{x}\rangle)$$

In the latter output $Lclm$ means the least common left multiple, i.e. the representation of the input ideal as the left intersection of three first order ideals. From this representation a fundamental system for the corresponding system of linear partial differential equations may be obtained. To this end, the differential equations for an unknown function z are defined first.

```
sys:={Df(z,y,2)+Df(z,y)/y-z/y**2,
      Df(z,x,y)+Df(z,x)/y+Df(z,y)/x+z/(x*y),
      Df(z,x,2)+Df(z,x)/x-z/x**2};
```

The subsequent call

```
Solve(sys|LDFMOD(RATF Q,{z},{x,y},GRLEX)|);
```

yields the answer

The differential equations:
$z_{yy}+\frac{1}{y}z_y-\frac{1}{y^2}z=0, z_{xy}+\frac{1}{y}z_x+\frac{1}{x}z_y+\frac{1}{xy}z=0, z_{xx}+\frac{1}{x}z_x-\frac{1}{x^2}z=0$
Fundamental system:
$z=\{\frac{y}{x},\frac{x}{y},\frac{1}{xy}\}$

The Loewy decomposition of a single operator is shown next. To this end submit

```
L:=D(x,3)+(y+1)*D(x,2,y)+(1-1/x)*D(x,2)+(1-1/x)*(y+1)*D(x,y)
        -1/x*D(x)-1/x*(y+1)*D(y);
LoewyDecompositionL|LDO(RATF Q,{x,y},GRLEX)|;
```

It yields the answer

The given operator:
$L\equiv\partial_{xxx}+(y+1)\partial_{xxy}+(1-\frac{1}{x})\partial_{xx}+(1-\frac{1}{x})(y+1)\partial_{xy}-\frac{1}{x}\partial_x-\frac{1}{x}(y+1)\partial_y$
The Loewy Decomposition:
$L=(\partial_x-\frac{1}{x})Lclm(\partial_x+1,\partial_x+(y+1)\partial_y)$

The solution of the corresponding partial differential equation is obtained by submitting

```
deq:=Df(z,x,3)+(y+1)*Df(z,x,2,y)+(1-1/x)*Df(z,x,2)+(1-1/x)*(y+1)*Df(z,x,y)
        -1/x*Df(z,x)-1/x*(y+1)*Df(z,y);
```

The given differential equation:
$z_{xxx}+(y+1)z_{xxy}+(1-\frac{1}{x})z_{xx}+(1-\frac{1}{x})(y+1)z_{xy}-\frac{1}{x}z_x-\frac{1}{x}(y+1)z_y=0$
The solution:
$z=F(y)e^{-x}+G((y+1)e^{-x})+(x+1)e^{-x}\int H(y)(y+1)dy$

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