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Solving Second Order Ordinary Differential Equations with Maximal Symmetry Group

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Abstract

Second order ordinary differential equations of the form $y'' + Ay'^3 + By'^2 + Cy' + D$ with $y \equiv y(x)$ and A, B, C and D functions of x and y are of special interest because they may allow the largest possible group of point symmetries if its coefficients satisfy certain constraints. For large classes of these equations a solution algorithm is described that determines its general solution in closed form by reducing it to a linear third-order equation. If the results obtained by Sophus Lie in the last century are supplemented by more recent concepts like Janet bases and Loewy decompositions, a systematic solution procedure is obtained that is easily implemented in a computer algebra system.

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1. Introduction

The subject of this article are second order ordinary differential equations (ode's) of the form

$$y'' + A(x, y)y'^3 + B(x, y)y'^2 + C(x, y)y' + D(x, y) = 0 \qquad (1)$$

with $A, B, C, D \in \mathbf{Q}(x, y)$, the *base field* of (1). These equations allow the eight-parameter projective group of the (x, y)—plane as symmetry group if its coefficients satisfy the constraints

$$\begin{aligned}\phi_1 \equiv {} & D_{yy} + BD_y - AD_x + (B_y - 2A_x)D + \frac{1}{3}B_{xx} - \frac{2}{3}C_{xy} \\ & + \frac{1}{3}C(B_x - 2C_y) = 0, \\ \phi_2 \equiv {} & 2AD_y + A_yD + \frac{1}{3}C_{yy} - \frac{2}{3}B_{xy} + A_{xx} - \frac{1}{3}BC_y \\ & + \frac{2}{3}BB_x - A_xC - AC_x = 0.\end{aligned} \qquad (2)$$