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Equivalence Classes, Symmetries and Solutions of Linear Third-order Differential Equations

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Abstract

The subject of this article are third-order differential equations that may be linearized by a variable change. To this end, at first the equivalence classes of linear equations are completely described. Thereafter it is shown how they combine into symmetry classes that are determined by the various symmetry types. An algorithm is presented allowing it to transform linearizable equations by hyper-exponential transformations into linear form from which solutions may be obtained more easily. Several examples are worked out in detail.

AMS Subject Classifications: 34A05, 34A25, 34B30, 68Q40.

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1. Introduction

It is the purpose of this article to describe solution algorithms for third order ordinary differential equations (ode's) that are equivalent to a linear one. This means, there exists a change of variables transforming the originally given equation into linear form. In order to utilize this property for the solution procedure, at first criteria are obtained allowing it to decide whether a given equation shares this property. Secondly for large classes of equations it is shown how the transformation to linear form is achieved algorithmically. Several results on linear ode's that are applied in this article may be found in Schlesingers' book [12] and some publications by Neumer [9], [10].

The general linear homogeneous equation will be written in the form:

$$y^{(n)} + q_1 y^{(n-1)} + \cdots + q_{n-1} y' + q_n y = 0, \tag{1}$$

where the coefficients $q_k \in \mathbf{Q}(x)$, i.e. the base field are the rational functions of the independent variable. The most general point-transformation preserving linearity and homogeneity is $x = F(u)$, $y = G(u)v$ with u and $v(u)$ the new independent and dependent variable respectively. These transformations are called the *structure*