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Factoring and Solving Linear Partial Differential Equations

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Abstract

The problem of factoring a linear partial differential operator is studied. An algorithm is designed which allows one to factor an operator when its symbol is separable, and if in addition the operator has enough right factors then it is completely reducible. Since finding the space of solutions of a completely reducible operator reduces to the same for its right factors, we apply this approach and execute a complete analysis of factoring and solving a second-order operator in two independent variables. Some results on factoring third-order operators are exhibited.

AMS Subject Classifications: 35A25, 35C05, 35G05.

Keywords: Partial differential equations, factorization.

1. Introduction

An algorithm for factoring a linear *ordinary* differential operator $L \in \mathbf{Q}(x)[\partial_x]$ is described in [15], it was improved in [5] with a better complexity bound. In [17] an implementation in a computer algebra system is given. A survey on the factorization problem and further references may be found in [14]. The structure of all possible factorizations of an *ordinary* differential operator is known due to a fundamental theorem of Loewy [12]. An ordinary operator has a *unique* factorization into completely reducible factors. Factorization of an operator L is of practical importance because it reduces the problem of finding solutions of the linear differential equation $Lv = 0$ to the same problem for its factors.

Much less is known on factoring linear *partial* differential operators (LPDO). First of all, the concept of a completely reducible operator has to be generalized suitably. In the articles by Blumberg [1] and Miller [13] these problems are discussed, and are illustrated by a few typical examples. In particular, an example of a third-order operator is given in [1] which has two different factorizations into completely reducible factors, see Example 4 in Sect. 4 for a complete discussion. It shows that the result of Loewy quoted above does not remain true for partial differential operators.

In recent times, due to the growing interest in Computer Algebra, a few papers have appeared which treat factoring as finding superideals of a left ideal in the