



3

3

Ideal intersections in rings of partial differential operators[☆]

Fritz Schwarz

Fraunhofer Gesellschaft, Institut SCAI, 53754 Sankt Augustin, Germany

ARTICLE INFO

Article history:

Received 7 May 2009

Accepted 7 April 2010

Available online xxxx

MSC:

35A25

35C05

35G05

Keywords:

Differential algebra

Ring of differential operators

Linear partial differential equations

ABSTRACT

An important step in solving linear differential equations in closed form is its factorization and generating the Loewy decomposition from it. For ordinary equations this is fairly straightforward because all operators involved generate principal ideals in the corresponding ring of operators. This is different for linear partial differential equations and the operators associated with them; its so-called non-unique factorizations have created some confusion in the past. Fundamental to Loewy's theory is the concept of a completely reducible operator; it is defined to be the left intersection of its irreducible constituents. Consequently a systematic investigation of the intersection ideals in the corresponding rings of differential operators is an indispensable requirement for generalizing Loewy's theory. The article at hand gives a thorough description of the possible intersection ideals of two first-order operators in two or three variables. Furthermore, it is shown how software provided on the website www.alltypes.de may be applied for solving concrete problems.

© 2010 Elsevier Inc. All rights reserved.

1. Introduction

The interest in the subject of this article originates from the factorization of differential operators and its corresponding Loewy-decomposition. For ordinary operators these factorizations have been established by Beke [1], Schlesinger [19], Loewy [14] and Ore [15]; a detailed discussion of second- and third-order operators may be found in Chapter 2 of the book by Schwarz [20]. At the core of Loewy's theory is the concept of a completely reducible operator; by definition an operator is completely reducible if it may be expressed as least common left multiple or left intersection of irreducible op-

[☆] Partially supported by the Austrian Science Fund No. P20336-N18, Project DIFFOP.

E-mail address: fritz.schwarz@scai.fhg.de.

URL: <http://www.scai.fhg.de/schwarz.html>.