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Factoring systems of linear PDEs with finite-dimensional solution spaces

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Abstract

A D-finite system is a finite set of linear homogeneous partial differential equations in several independent and dependent variables, whose solution space is of finite dimension. Let L be a D-finite system with rational function coefficients. We present an algorithm for computing all hyperexponential solutions of L, and an algorithm for computing all D-finite systems whose coefficients are also rational functions, and whose solutions are contained in the solution space of L.

1. Introduction

For various reasons *linear* differential equations have been of particular importance in the history of mathematics. First of all, the problems connected with them are much easier than those for nonlinear equations. Second, many nonlinear problems may be linearized in some way such that the results of the former theory may be applied to them. This is especially true for Lie's symmetry analysis of ordinary differential equations (ODEs) which reduces the problem of solving nonlinear ODEs with a sufficiently large number of symmetries to the study of certain systems of linear partial differential equations (PDE's). The problem of finding conservation laws for nonlinear PDE's also leads to systems of linear PDE's.

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