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Solving Inhomogeneous Linear Partial Differential Equations

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Abstract. Lagrange's variation-of-constants method for solving linear inhomogeneous ordinary differential equations (ode's) is replaced by a method based on the Loewy decomposition of the corresponding homogeneous equation. It uses only properties of the equations and not of its solutions. As a consequence it has the advantage that it may be generalized for partial differential equations (pde's). It is applied to equations of second order in two independent variables, and to a certain system of third-order pde's. Therewith all possible linear inhomogeneous pde's are covered that may occur when third-order linear homogeneous pde's in two independent variables are solved.

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1 Introduction

Linear differential equations have been considered extensively in the mathematical literature, beginning in the second half of the 19th century. For linear homogeneous ordinary differential equations (ode's) there exists a fairly complete theory, culminating in differential Galois theory and algorithms for finding large classes of solutions. Here this means always a closed form solution in some well defined function space; in particular numerical or graphical solutions are excluded. For inhomogeneous equations, Lagrange's method of variation-of-constants allows finding a special solution if a fundamental system for the homogeneous equation is known.

For linear partial differential equations (pde's) the answer is much less complete. For homogeneous equations factorizations and Loewy decompositions appear to be the best

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