Symmetry Analysis of Abel's Equation

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A solution algorithm for Abel's equation and some generalizations based on a nontrivial Lie symmetry of a particular kind, i.e., so-called *structure-preserving symmetry*, is described. For the existence of such a symmetry a criterion in terms of the coefficients of the so-called *rational normal form* of the given equation is derived. If it is affirmative, solving Abel's equation is reduced to a well-defined integration problem. It is shown that almost all known ad hoc methods for obtaining closed form solutions are consequences of this type of symmetry. Possible extensions of this scheme to more general classes of first-order ordinary differential equations are pointed out.

1. Introduction

Solving nonlinear ordinary differential equations (ODEs) is dominated to a large extent by various ad hoc methods as may be seen from the collections by Kamke [1] and Murphy [2]. The main deficiencies of these database-like approaches are well known. If an equation is not used exactly as given, it is almost useless in most cases. Even worse, if an equation cannot be solved by applying these collections or the various ad hoc methods described there, it is by no means guaranteed that a closed form solution does not exist. The

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